**Convexity**

Table of Contents

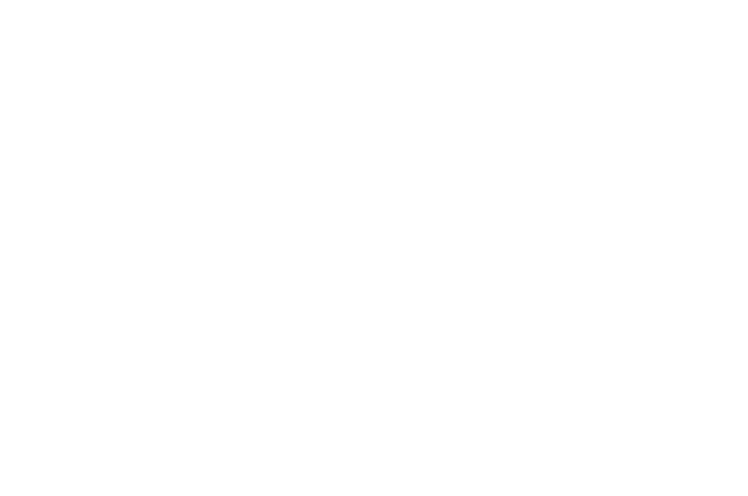
[Convex Sets and Convex Functions 2](#_Toc100149574)

[Proving Cost Functions are Convex 2](#_Toc100149575)

## Convex Sets and Convex Functions

Consider a set of points from which we randomly select two and connect them using a line. Lets say that all possible points on that line fall within that same set. If this is true for all pairs of points in that set, this is called a **convex set**.

Our main interest however, is with **convex functions**. A convex function is one in which line segments between any given pair of points from the function lie above the graph for the function. The **domain** for such a function is a **convex set**.



## Proving Cost Functions are Convex

Our main goal at the moment is to prove that all **cost functions** are **convex functions**. As we will later see, the **vectorized format** of the cost function can be written as:

If we differentiate this twice, we get:

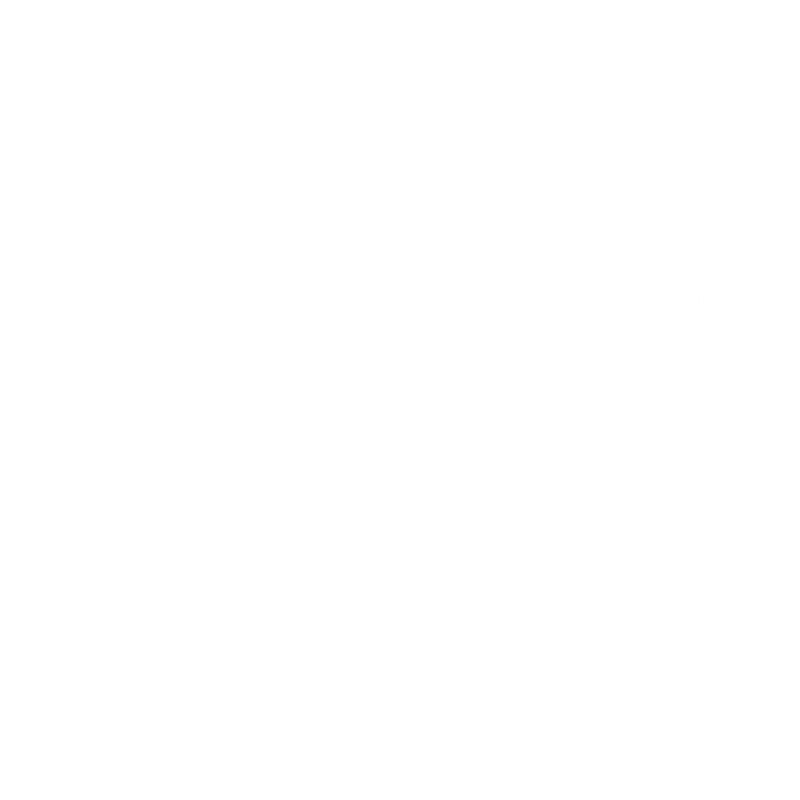
- ()

From **Taylor’s Theorem**, we know that:

Since we know that condition (i) is true, the term In Taylor’s Theorem must be a positive value. If we remove this term (and every term after it), we get:

- ()

This statement must be true for all **convex functions**. We can prove this graphically.



At the point , we have the values and . Extending the tangent at , the value of the tangent at is . However, the value of is greater than this value. Thus,

If we simply consider a multi-dimensional case, then we will have instead of . Thus, all cost functions must be convex.